1. Find

$$\int \frac{3x^4 - 4}{2x^3}\, dx$$

writing your answer in simplest form.

(4)

$$\int \frac{3x^4 - 4}{2x^3}\, dx = \int \frac{3x^4}{2x^3} - \frac{4}{2x^3}\, dx \quad ①$$

$$= \int \frac{3}{2}x - 2x^{-3}\, dx \quad ①$$

$$= \frac{3}{2} \times \frac{x^2}{2} - 2 \times \frac{x^{-2}}{2} + c \quad ①$$

$$= \frac{3}{4}x^2 + \frac{1}{x^2} + c \quad ①$$

2. Find

$$\int \left(8x^3 - \frac{3}{2\sqrt{x}} + 5\right) dx$$

giving your answer in simplest form.

(4)

$$\int \left(8x^3 - \frac{3}{2}x^{-\frac{1}{2}} + 5\right) dx = \frac{1}{4} \times 8x^4 + 2 \times -\frac{3}{2}x^{\frac{1}{2}} + 5x + c$$ ① ① ①

$$= 2x^4 - 3x^{\frac{1}{2}} + 5x + c$$ ①

Integration : $\int ax^b = \frac{ax^{b+1}}{b+1} + c$

3. The height, h metres, of a plant, t years after it was first measured, is modelled by the equation

$$h = 2.3 - 1.7e^{-0.2t} \qquad t \in \mathbb{R} \quad t \geqslant 0$$

Using the model,

(a) find the height of the plant when it was first measured,

(2)

(b) show that, exactly 4 years after it was first measured, the plant was growing at approximately 15.3 cm per year.

(3)

According to the model, there is a limit to the height to which this plant can grow.

(c) Deduce the value of this limit.

(1)

a) t years = years after first measured

first measured = 0 years

so, height when plant was first measured :

$h = 2.3 - 1.7e^{-0.2(0)}$ (1)

$= 2.3 - 1.7 = 0.6$ m (1)

calculating rate of change of height of the plant

b) $\frac{dh}{dt} = 0.34e^{-0.2t}$ (1)

when $t = 4$,

$\frac{dh}{dt} = 0.34e^{-0.2(4)}$ (1)

$= 0.153$ m ≈ 15.3 cm (1)

c) when t approaching ∞, the height will be 2.3 m. (1)

2.3 m is the value of the limit.

4. A curve has equation $y = f(x), \; x \geqslant 0$

Given that

- $f'(x) = 4x + a\sqrt{x} + b$, where a and b are constants
- the curve has a stationary point at $(4,3)$
- the curve meets the y-axis at -5

find $f(x)$, giving your answer in simplest form.

(6)

when curve is at stationary point, $f'(x) = 0$, $x = 4$ and $y = 3$.

$f'(x) = 4x + a\sqrt{x} + b$

$0 = 4(4) + a\sqrt{4} + b$

$0 = 16 + 2a + b$ (1) , $b = -2a - 16$ — (1)

To get $f(x)$, we will integrate $f'(x)$.

$f'(x) = 4x + a\sqrt{x} + b$ (1)

$f(x) = 2x^2 + \frac{2}{3}ax^{\frac{3}{2}} + bx + c$ (1)

y-intercept is -5, so the value of $c = -5$ (1)

$f(x) = 2x^2 + \frac{2}{3}ax^{\frac{3}{2}} + bx - 5$

from the stationary point, we know that $f(4) = 3$

$3 = 2(4)^2 + \frac{2}{3}a(4)^{\frac{3}{2}} + 4b - 5$

$3 = 32 + \frac{16}{3}a + 4b - 5$

$4b = -24 - \frac{16}{3}a$

$b = -6 - \frac{16}{12}a$ — (2)

substitute 1 into 2

$-2a - 16 = -6 - \frac{16}{12}a$

$-2a + \frac{4}{3}a = 10$

$-\frac{2}{3}a = 10$

$a = -15 \quad , \quad b = 14$ 1

$f(x) = 2x^2 + \frac{2}{3}(-15)x^{3/2} + 14x - 5$

$= 2x^2 - 10x^{\frac{3}{2}} + 14x - 5$ 1

5. Find

$$\int \frac{x^{\frac{1}{2}}(2x-5)}{3}\,dx$$

writing each term in simplest form.

(4)

$x^{1/2}(2x-5) = 2x^{3/2} - 5x^{1/2}$ ① $\leftarrow x^a \times x^b = x^{a+b}$

$\int \frac{2}{3}x^{3/2} - \frac{5}{3}x^{1/2}\,dx$ ①

$= \frac{2}{5} \times \frac{2}{3}x^{5/2} - \frac{2}{3} \times \frac{5}{3}x^{3/2} + C$ ①

$= \frac{4}{15}x^{5/2} - \frac{10}{9}x^{3/2} + C$ ①

6. The curve C has equation $y = \mathrm{f}(x)$

The curve

- passes through the point $P(3, -10)$
- has a turning point at P

Given that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x^3 - 9x^2 + 5x + k$$

where k is a constant,

(a) show that $k = 12$

(2)

(b) Hence find the coordinates of the point where C crosses the y-axis.

(3)

a) $\left.\frac{dy}{dx}\right|_{x=3} = 0$ because P is a turning point

$2(3)^3 - 9(3)^2 + 5(3) + k = 0$ 1

$54 - 81 + 15 + k = 0 \Rightarrow k = 12$ 1

b) $y = \int \frac{dy}{dx}\,dx = \int (2x^3 - 9x^2 + 5x + 12)\,dx$

$y = \frac{1}{2}x^4 - 3x^3 + \frac{5}{2}x^2 + 12x + c$ 1

sub in $x = 3, y = -10$

$-10 = \frac{1}{2}(3)^4 - 3(3)^3 + \frac{5}{2}(3)^2 + 12(3) + c$ 1

$c = -28$

$y = \frac{1}{2}x^4 - 3x^3 + \frac{5}{2}x^2 + 12x - 28$

C crosses y-axis when $x = 0 \therefore y = -28$

$(0, -28)$ 1